

# 1 Introduction to Systems Modeling Concepts

(Chapter 1 from Zeigler, B. P., H. Praehofer and T. G. Kim (2000). Theory of Modeling and Simulation: Integrating Discrete Event and Continuous Complex Dynamic Systems, (2<sup>nd</sup> Ed.) Academic Press, NY.)

This chapter introduces some key concepts that underlie the framework and methodology for modeling and simulation (M&S) originally presented in “Theory of Modeling and Simulation” published in 1976 – referred to hereafter as TMS76 to distinguish it from the current revised edition TMS98. Perhaps the most basic concept is that of mathematical systems theory. First developed in the nineteen sixties, this theory provides a fundamental, rigorous mathematical formalism for representing dynamical systems. There are two main, and orthogonal, aspects to the theory:

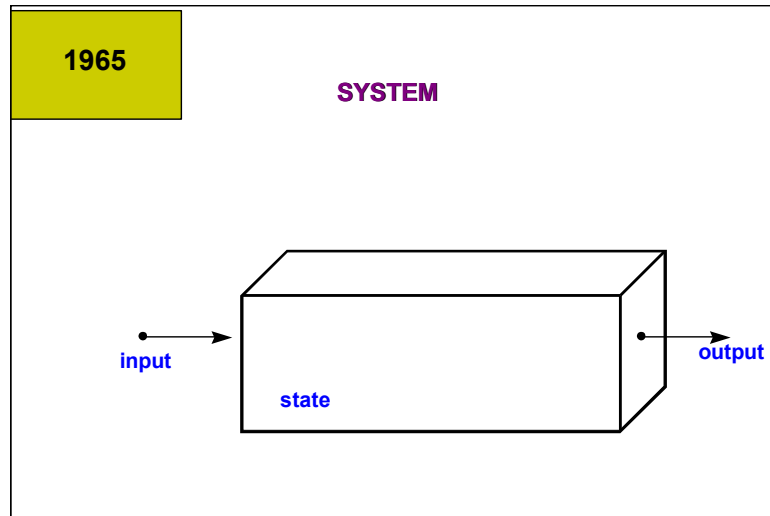
- ◆ **levels of system specification** – these are the levels at which we can describe how systems behave and the mechanisms that make them work the way they do.
- ◆ **systems specification formalisms** – these are the types of modeling styles, such continuous or discrete, that modelers can use to build system models.

Although the theory is quite intuitive, it does present an abstract way of thinking about the world that you will probably find unfamiliar. So we introduce the concepts in a spiral development consisting of easy-to-grasp stages – with each spiral revolution returning to a more faithful version of the full story.

In this chapter we first introduce some basic systems concepts, then motivate the systems specification formalisms by describing their evolution over time. This also provides a way to point out the differences between TMS98 and its predecessor. Finally, we discuss the levels of system specification, illustrating them with familiar examples. In this ground stage of our spiral development, the presentation is informal and prepares the way for the framework for M&S that comes in the next chapter. Later, in the second part of the book, we return to a more rigorous development of the concepts to lay a sound basis for the developments to come in the third part.

## 1.1 Systems Specification Formalisms

System theory distinguishes between system *structure* (the inner constitution of a system) and *behavior* (its outer manifestation). Viewed as a black box (Figure 1) the external *behavior* of a system is the relationship it imposes between its input time histories and output time histories. The system’s input/output behavior consists of the pairs of data records (input time segments paired with output time segments) gathered from a real system or model. The internal *structure* of a system includes its state and state transition mechanism (dictating how inputs transform current states into successor states) as well as the state-to-output mapping. Knowing the system structure allows us to deduce (analyze, simulate) its behavior. Usually, the other direction (inferring structure from behavior) is not univalent – indeed, discovering a valid representation of an observed behavior is one of the key concerns of the M&S enterprise.



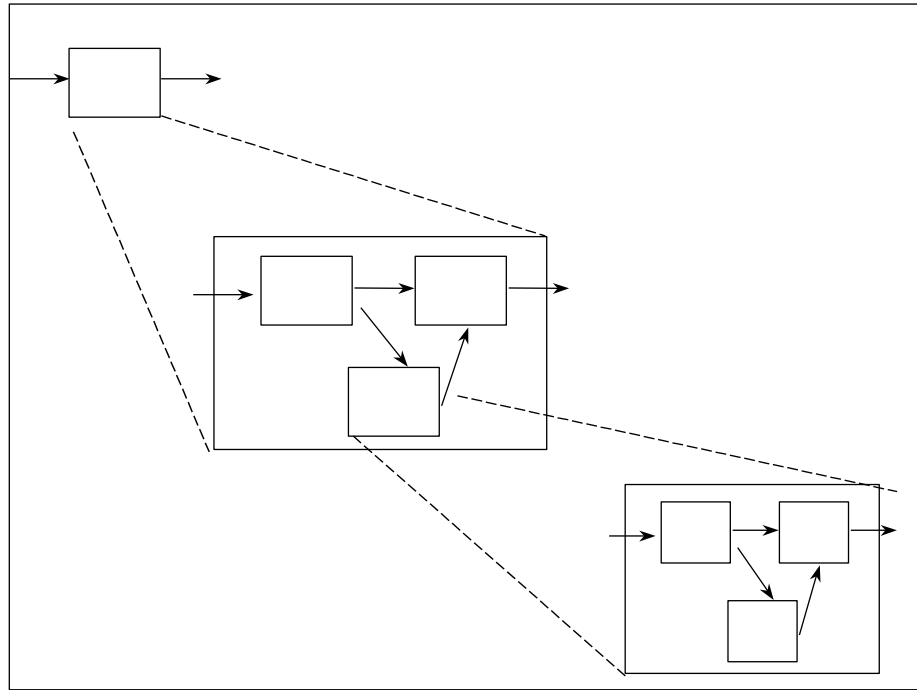
**Figure 1 Basic System Concepts**

An important structure concept is that of *decomposition* namely, how a system may be broken down into component systems (Figure 2). A second concept is that of *composition*, i.e., how component systems may be coupled together to form a larger system. Systems theory is *closed under composition* in that the structure and behavior of a composition of systems can be expressed in the original system theory terms. The ability to continue to compose larger and larger systems from previously constructed components leads to *hierarchical construction*. Closure under composition guarantees that such a composition results in a system, called its *resultant*, with well defined structure and behavior. *Modular* systems have recognized input and output ports through which all interaction with the environment occurs. They can be *coupled* together by coupling output ports to input ports and can have hierarchical structure in which component systems are coupled together to form larger ones.

The difference between a decomposed systems, as in Figure 2, and undecomposed systems, as in Figure 1, provides our first introduction to *levels* of systems specification. We'll say later that the former are at a higher level of specification than the latter since they provide more information about the structure of the system.

### 1.1.1 Relation To Object Orientation

Models developed in a system theory paradigm bear a resemblance to concepts of object-oriented programming. Both *objects* and system models share a concept of internal state. However, mathematical systems are formal structures that operate on a time base while programming objects typically do not have an associated temporal semantics. Objects in typical object oriented paradigms are not hierarchical or modular in the sense just described. The coupling concept in modular systems provides a level of delayed binding – a system model can place a value on one of its ports but the actual destination of this output is not determined until the model becomes a component in a larger system and a coupling scheme is specified. It can therefore: a) be developed and tested as a stand alone unit, b) be placed in a model repository and reactivated at will and c) reused in any applications context in which its behavior is appropriate and coupling to other components makes sense.



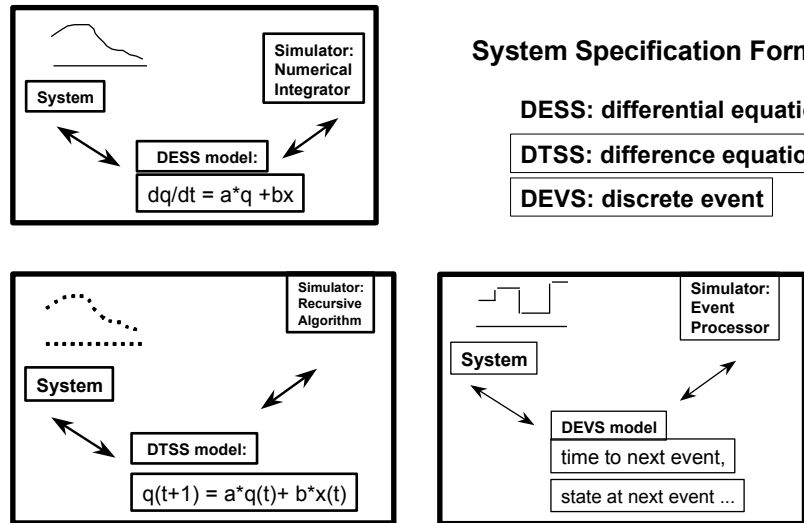
**Figure 2 Hierarchical System Decomposition**

While coupling establishes output-to-input pathways, the systems modeler is completely free to specify how data flows along such channels. Information flow is one of many interactions that may be represented. Other interactions include physical forces and fields, material flows, monetary flows, and social transactions. The systems concept is broad enough to include the representation of any of these and supports the development of M&S environments that can make including many within the same large-scale model.

Although systems models have formal temporal and coupling features not shared by conventional objects, object-orientation does provide a supporting computational mechanism for system modeling. Indeed, there have been many object-oriented implementations of hierarchical, modular modeling systems. These demonstrate that object-oriented paradigms, particularly for distributed computing, can serve as a strong foundation to implement the modular systems paradigm.

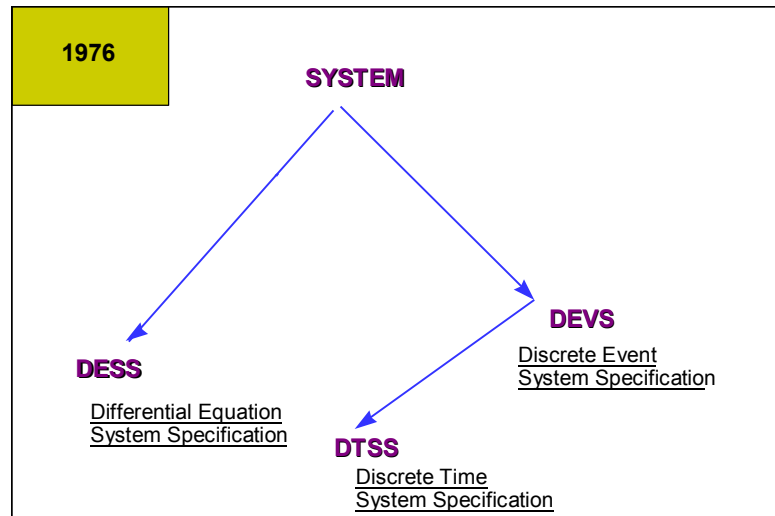
### 1.1.2 Evolution Of Systems Formalisms

As in many situations, portraying the evolution of an idea may help in the understanding of the complexities as they develop. 0 depicts the basic systems modeling formalisms as they were presented in the first edition, TMS76. This edition was the first book to formulate approaches to modeling as system specification formalisms – shorthand means of delineating a particular system within a subclass of all systems. The traditional differential equation systems, having continuous states and continuous time, were formulated as the class of DESS (Differential Equation System Specifications). Also, systems that operated on a discrete time base such as automata were formulated as the class of DTSS (Discrete Time System Specifications). In each of these cases, mathematical representation had preceded their computerized incarnations (it has been three hundred years since Newton-Leibnitz!).



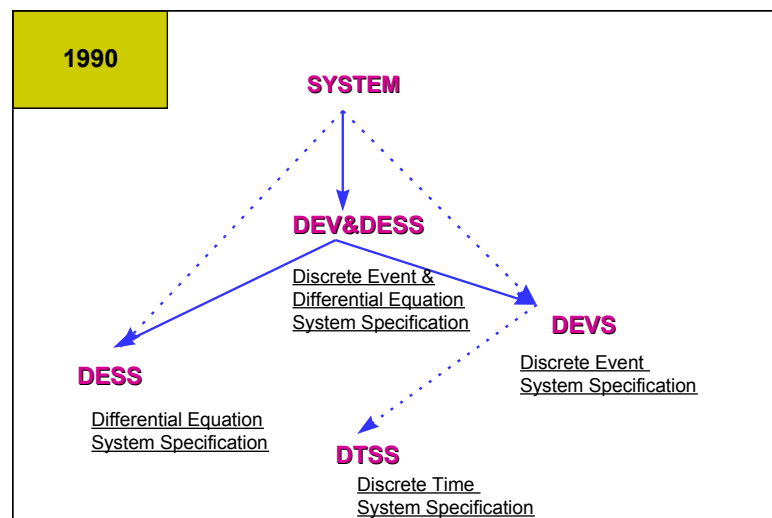
**Figure 3 Basic Systems Specification Formalisms**

However, the reverse was true for the third class, the Discrete Event System Specifications (DEVS). Discrete event models were largely prisoners of their simulation language implementations or algorithmic code expressions. Indeed, there was a prevalent belief that discrete event “world views” constituted new mutant forms of simulation, unrelated to the traditional mainstream paradigms. Fortunately, that situation has begun to change as the benefits of abstractions in control and design became clear. Witness the variety of discrete event dynamic system formalisms that have emerged [Ho 94]. While each one – examples are Petri Nets, Min-Max algebra, and GSMP (generalized semi-markov processes) – has its application area, none were developed deliberately as subclasses of the systems theory formalism. Thus to include such a formalism into an organized system-theory based framework requires “embedding” it into DEVS.



**Figure 4 The Dynamics of Basic System Classes**

“Embedding.” What could such a concept mean? The arrows in Figure 4 indicate *subclass* relationships; for example, they suggest that DTSS is a “subclass of” DEVS. However, it is not literally true that any discrete time system is also discrete event system (their time bases are distinct, for example). So we need a concept of *simulation* that allows us to say when one system can do the essential work of another. One formalism can be *embedded* in another if any system in the first can be simulated by some system in the second. Actually, more than one such relationship, or *morphism*, may be useful, since, as already mentioned, there are various levels of structure and behavior at which equivalence of systems could be required. As a case in point, the TMS76 edition established that any DTSS could be simulated by a DEVS by constraining the time advance to be constant. However, this is not as useful as it could be until we can see how it applies to *decomposed* systems. Until that is true, we either must reconstitute a decomposed discrete time system to its resultant before representing it as a DEVS or we can represent each DTSS component as a DEVS but we can’t network the DEVS together to simulate the resultant. The current edition, TMS98, establishes this stronger simulation relation (Chapter 15).



**Figure 5 Introducing the DEV&DESS Formalism**

### 1.1.3 Combining Continuous And Discrete Formalisms

Skipping many years of accumulating developments, the next major advance in systems formalisms was the combination of discrete event and differential equation formalisms into one, the DEV&DESS. As shown in Figure 5, this formalism subsumes both the DESS and the DEVS (hence also the DTSS) and thus supports the development of coupled systems whose components are expressed in any of the basic formalisms. Such *multiformalism* modeling capability is important since the world does not usually lend itself to using one form of abstraction at a time. For example, a chemical plant is usually modeled with differential equations while its control logic is best designed with discrete event formalisms. In 1990, one of the authors of this book showed that DEV&DESS was closed under coupling and in order to do so, had to deal with the pairs of input-output interfaces between the different types of systems. Closure under coupling also required that the DEV&DESS formalism provide a means to specify components with intermingled discrete and continuous expressions. Finally, simulator algorithms (so called *abstract simulator*) had to be provided to establish that the new formalism could be implemented in computational form (look ahead to Chapter 9 to see how this was all accomplished).

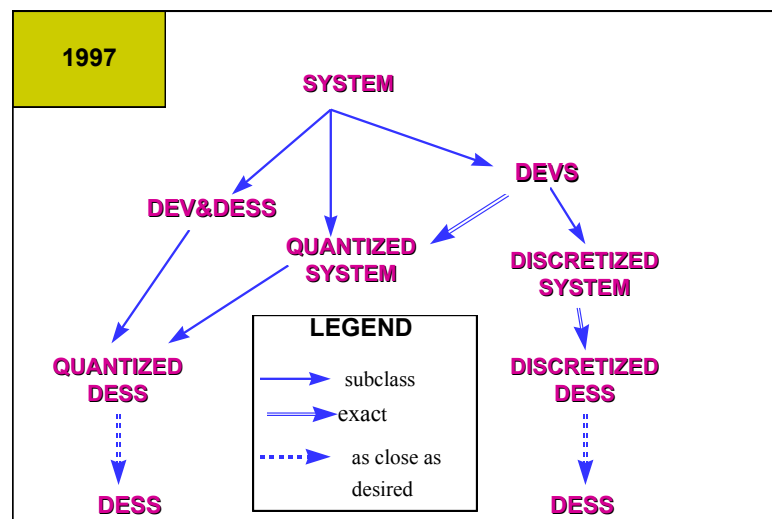


Figure 6 Introducing Quantized Systems

### 1.1.4 Quantized Systems

The current edition, TMS98, builds on the advances since 1976 especially in the directions pointed to by the introduction of DEV&DESS. Since parallel and distributed simulation is fast becoming the dominant form of model execution, and discrete event concepts best fit with this technology, our focus is on a concept called the *DEVS bus*. This concept, introduced in 1996 by another of the authors and his students, concerns the use of DEVS models, as a “wrappers” to enable a variety of models, to interoperate in a networked simulation. It is particularly germane to the High Level Architecture (HLA) defined by the United States Department of Defense. One way of looking at this idea is that we want to embed any formalism, including for example, the DEV&DESS, into DEVS. Another way is to introduce a new class of systems, called the Quantized System, as illustrated in Figure 6. In such systems, both the input and output are quantized. As an example, an analog-to-digital converter does such quantization by mapping a real number into a finite string of digits. In general, quantization forms equivalence classes of outputs that then become indistinguishable for downstream input receivers, requiring less data network bandwidth, but also possibly incurring error.

Quantization provides a process for representing and simulating continuous systems that is an alternative to the more conventional discretization of the time axis. While discretization leads to discrete time systems, quantization leads to discrete event systems. The beginnings of a theory of quantized systems are presented in Chapter 16 of this book. When we restrict quantization to differential equation systems, we can express the resulting class, Quantized DESS, within DEV&DESS and study its properties, especially from the point of view of the DEVS bus. We can then study the approximation capability and simulation efficiency of DEVS in distributed simulation in comparison with classical time stepped integration and simulation approaches. Particularly with respect to reduction of message passing and network bandwidth (a major concern in distributed simulation) promising results are being obtained.

### 1.1.5 Extensions of DEVS

Various extensions of DEVS have been developed as illustrated in Figure 7. They will be discussed in Chapter 10. These developments expand the classes of system models that can be represented in DEVS, and hence, integrated within both the DEVS bus and the parent systems theory formalism.

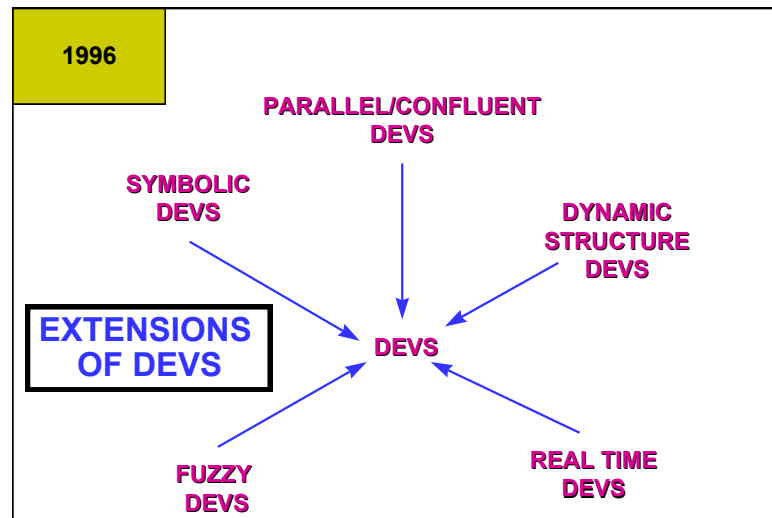
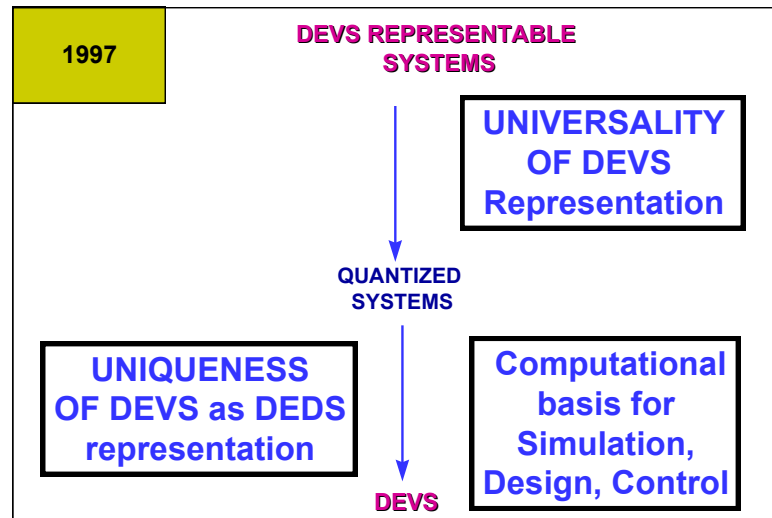


Figure 7 Extensions of the DEVS Formalism

These developments lend credence to the claim that DEVS is a promising computational basis for analysis and design of systems, particularly when simulation is the ultimate environment for development and testing (Figure 8). The claim rests on the *universality* of the DEVS representation, namely the ability of DEVS bus to support the basic system formalisms. This edition goes some distance toward substantiating the claim that DEVS is the unique form of representation that underlies any system with discrete event behavior. This *uniqueness* claim of DEVS, discussed in Chapter 15, offers the promise that the profusion of discrete event formalisms under development for control and management of systems can be embedded as subformalisms of DEVS in the DEVS bus and thus made accessible in an integrated distributed simulation environment.



**Figure 8** DEVS as a Computational Basis for Simulation, Design and Control

## 1.2 Levels of System Knowledge

As already mentioned, the systems specification hierarchy is the basis for a framework for M&S which sets forth the fundamental entities and relationships in the M&S enterprise. The hierarchy is first presented in an informal manner and later in Chapter 5 in its full mathematical rigor. Our presentation starts with a review of George Klir's [Klir 1985] systems framework.

Level	Name	What we know at this level
0	Source	what variables to measure and how to observe them
1	Data	data collected from a source system
2	Generative	means to generate data in a data system
3	Structure	components (at lower levels) coupled together to form a generative system

**Table 1** Levels of System Knowledge.

Table 1 identifies four basic levels of knowledge about a system recognized by Klir. At each level we know some important things about a system that we didn't know at lower levels. At the lowest level, the *source level* identifies a portion of the real world that we wish to model and the means by which we are going to observe it. As the next level, the *data level* is a data base of measurements and observations made for the source system. When we get to Level 2, we have the ability to recreate this data using a more compact representation, such as a formula. Since typically, there are many formulas or other means to generate the same data, the *generative level*, or particular means or formula we have settled on, constitutes knowledge we didn't have at the data system level. When people talk about models in the context of simulation studies they are usually referring to the concepts identified at this level. That is, to them a model means a program to generate data. At the last level, the *structure level*, we have a very specific kind of generative system. In other words, we know how to generate the data observed at Level 1 in a more specific manner – in terms of component systems that are interconnected together



and whose interaction accounts for the observations made. When people talk about systems, they are often referring to this level of knowledge. They think of reality as being made up of interacting parts – so that the whole is the sum (or a sometimes claimed, more, or less, than the sum) of its parts. Although some people use the term ‘subsystems’ for these parts, we call them **component** systems (and reserve the term subsystem for another meaning).

As we have suggested, Klir’s terms are by no means universally known, understood, or accepted in the M&S community. However, his framework is a useful starting point since it provides a unified perspective on what are usually considered to be distinct concepts. From this perspective, there are only three basic kinds of problems dealing with systems and they involve moving between the levels of system knowledge (Table 2). In *systems analysis*, we are trying to understand the behavior of an existing or hypothetical system based on its known structure. *Systems inference* is done when we don’t know what this structure is – so we try to guess this structure from observations that we can make. Finally, in *systems design*, we are investigating the alternative structures for a completely new system or the redesign of an existing one.

Systems Problem	Does source of the data exist? What are we trying to learn about it?	Which level transition is involved?
systems analysis	The system being analyzed may exist or may be planned. In either case we are trying to understand its behavioral characteristics.	moving from higher to lower levels, e.g., using generative information to generate the data in a data system
systems inference	The system exists. We are trying to infer how it works from observations of its behavior.	moving from lower to higher levels, e.g., having data, finding a means to generate it
systems design	The system being designed does not yet exist in the form that is being contemplated. We are trying to come up with a good design for it.	moving from lower to higher levels, e.g. having a means to generate observed data, synthesizing it with components taken off the shelf.

**Table 2 Fundamental Systems Problems.**

The central idea is that when we move to a lower level, we don’t generate any really new knowledge – we are only making explicit what is implicit in the description we already have. One could argue that making something explicit can lead to insight, or understanding, which is a form of new knowledge, but Klir is not considering this kind of subjective (or modeler dependent) knowledge. In the M&S context, one major form of *systems analysis* is computer simulation which generates data under the instructions provided by a model. While no new knowledge (in Klir’s sense) is generated, interesting properties may come to light of which we were not aware before the analysis. On the other hand, *systems inference* and *systems design* are problems that involve **climbing up** the levels. In both cases we have a low level system description and wish to come up with an equivalent higher level one. For systems inference, the lower level system is typically at the data system level, being data that we have observed from some existing source system. We are trying to find a generative system, or even a structure system, that can recreate the observed data. In the M&S context, this is usually called *model construction*. In the case of systems design, the source system typically does not yet exist and our objective is to build one that has a desired functionality. By functionality we mean what we want the system to do; typically, we want to come up with a structure system, whose components are technological, i.e., can be obtained off-the-shelf,

or built from scratch from existing technologies. When these components are interconnected, as specified by a structure system's coupling relation, the result should be a real system that behaves as desired.

It is interesting to note that the process called *reverse engineering* has elements of both inference and design. To reverse engineer an existing system, such as was done in the case of the cloning of IBM compatible PCs, an extensive set of observations is first made. From these observations, the behavior of the system is inferred and an alternative structure to realize this behavior is designed – thus bypassing patent rights to the original system design!

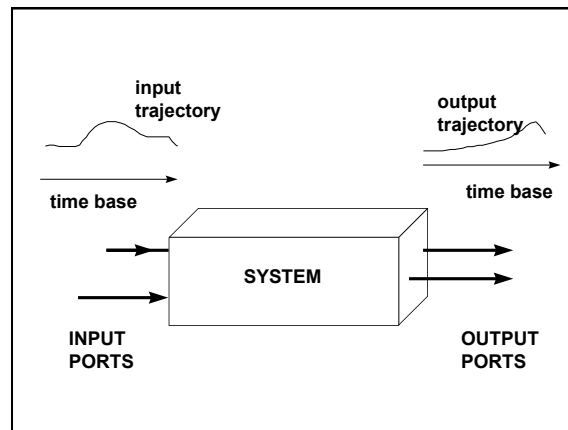
### 1.3 Introduction to the Hierarchy of Systems Specifications

At about the same time (in the early 1970's) that Klir introduced his epistemological (knowledge) levels, TMS76 formulated a similar hierarchy that is more oriented toward the M&S context. This framework employs a general concept of *dynamical system* and identifies useful ways in which such a system can be specified. These ways of describing a system can be ordered in levels as in Table 3. Just as in Klir's framework, at each level more information is provided in the specification that cannot be derived from lower levels. As can be seen in Table 3, these levels roughly correspond to those of Klir's framework.

Level	Specification Name	Corresponds to Klir's	What we know at this level
0	Observation Frame	Source System	how to stimulate the system with inputs; what variables to measure and how to observe them over a time base;
1	I/O Behavior	Data System	time-indexed data collected from a source system; consists of input/output pairs
2	I/O Function		knowledge of initial state; given an initial state, every input stimulus produces a unique output.
3	State Transition	Generative System	how states are affected by inputs; given a state and an input what is the state after the input stimulus is over; what output event is generated by a state.
4	Coupled Component	Structure System	components and how they are coupled together. The components can be specified at lower levels or can even be structure systems themselves – leading to hierarchical structure.

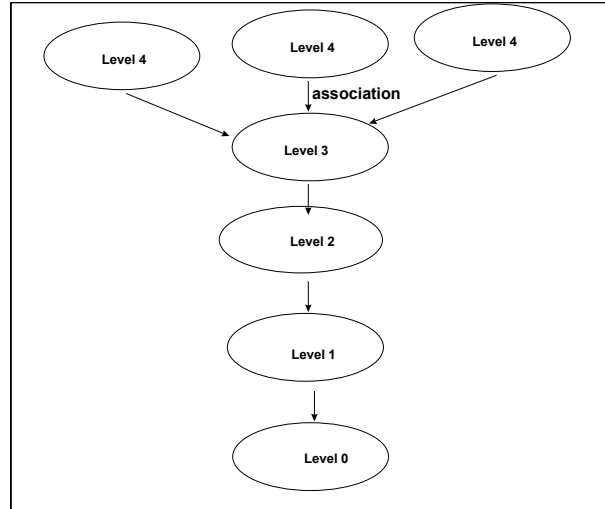
**Table 3 Relation between System Specification Hierarchy and Klir's levels.**

The major difference between the two frameworks is that the System Specification Hierarchy recognize that simulation deals with **dynamics**, the way in which systems behave over time. Therefore, *time* is the base upon which all events are ordered. We also view systems as having *input* and *output* interfaces through which they can interact with other systems. As illustrated in Figure 9, systems receive stimuli ordered in time through their *input ports*, and respond on their *output ports*. The term “port” signifies a specific means of interacting with the system. Whether by stimulating it (input) or observing it (output). The time-indexed inputs to systems are called *input trajectories*; likewise, their time-indexed outputs are called *output trajectories*. Ports are the only channels through which one can interact with the system. This means that system are *modular*. While Klir’s framework can include dynamics, input/output ports and modularity, it is not dedicated to these concepts. However, understanding these concepts is critical to effectively solving the problems that arise in M&S.



**Figure 9 Input/Output System.**

Before discussing each level of the specification hierarchy in some detail, let’s observe that we could have the very same real world object specified simultaneously at each of the levels. Thus there should be a way to associate the next lower level specification with any given one. This association concept is illustrated in Figure 10. For example, if we have know the detailed structure at the Coupled Component level, then we ought to be able to construct the corresponding specification at the State Transition level. The hierarchy is set up to provide such an *association mapping* at each (other than the lowest) level. Indeed, this is the formal version of climbing down the levels just discussed. Since the association mapping is not necessarily one to one, many upper level specifications may map to the same lower level one. This is the underlying reason why climbing up the levels is much harder than climbing down the levels. Indeed, when we select one of the associated upper level specifications for a given lower level one, we are gaining knowledge we didn’t have at the lower level.



**Figure 10 Association between levels of the System Specification Hierarchy**

## 1.4 The Specification Levels Informally Presented

### 1.4.1 Observation Frame

The Observation Frame specifies how to stimulate the system with inputs; what variables to measure and how to observe them over a time base.

As an example, Figure 11 shows a forest subject to *lightning*, *rain* and *wind*, modeled as input ports and *smoke* produced from fire, represented as an output port. This is a level 0 or Observation Frame specification. Note the choice of variables we included as ports and their *orientation* (i.e., whether they are input or output ports). We could have chosen differently. For example, we could have included an output port to represent heat radiation. Moreover, rather than representing each variable by a single value, it can be distributed over space, i.e., represented by an array of values. Such choices depend on our modeling objectives and are specified through *experimental frames*, a concept which we discuss in Chapter 2.

Figure 12 shows some examples of input and output trajectories. The input trajectory on the *lightning* port shows a bolt occurring at some particular time  $t_0$ . Only one such bolt occurs in the time period shown. The *smoke* output trajectory, at the top, depicts a gradual build up of smoke starting at  $t_0$  (so presumably, caused by a fire started by the lightning bolt). The possible values taken on by *smoke*, called its *range*, would result from some appropriate measurement scheme, e.g., measuring density of particulate material in grams/cubic meter. The pair of input, and associated output, trajectories is called a *input/output (or I/O) pair*. Figure 12 also displays a second I/O pair with the same input trajectory but different output trajectory. It represents the fact that there may be many responses to the same stimulus. In the second case, lightning did not cause a major fire, since the one that broke out quickly died. Such multiple output trajectories (for the same input trajectory) are characteristic of knowledge at Level 1. Knowing how to disambiguate these output trajectories is knowledge we will gain at the next level.

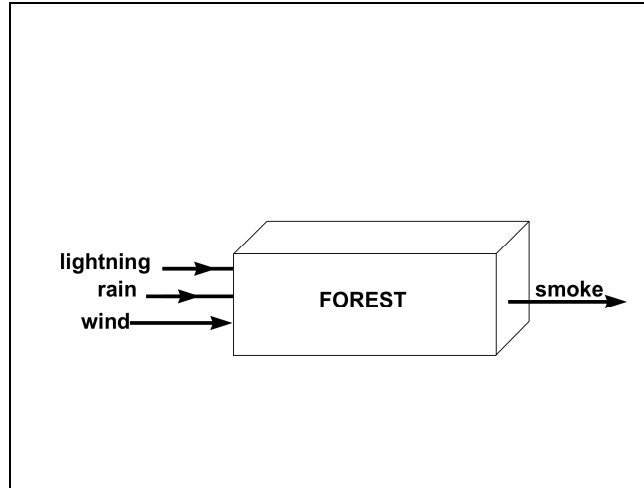


Figure 11 A forest specified as a system in the Observation Frame (Level 0)..

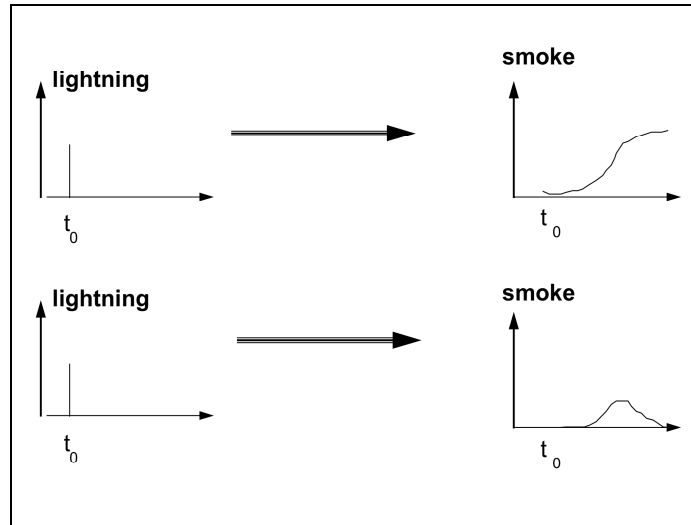


Figure 12 Some Input-Output Pairs for the Forest System Frame of Figure 11.

#### 1.4.2 I/O Behavior and I/O Function

The collection of all I/O pairs gathered by observation is called the *I/O Behavior* of a system. Returning to Table 3, this represents a system specification at Level 1. Now suppose that we are able to uniquely predict the response of the smoke output to a lightning bolt. For example, suppose we know that if the vegetation is dry, then a major fire will ignite, but if the vegetation is moist then any fire will quickly die. Having such a factor represents knowledge at Level 2, that of the *I/O Function*. Here, in addition to lower level information, we add *initial states* to the specification – when the initial state is known, there is a functional relationship between input and output trajectories, i.e., the initial state determines the **unique** response to any input (Figure 13 a).

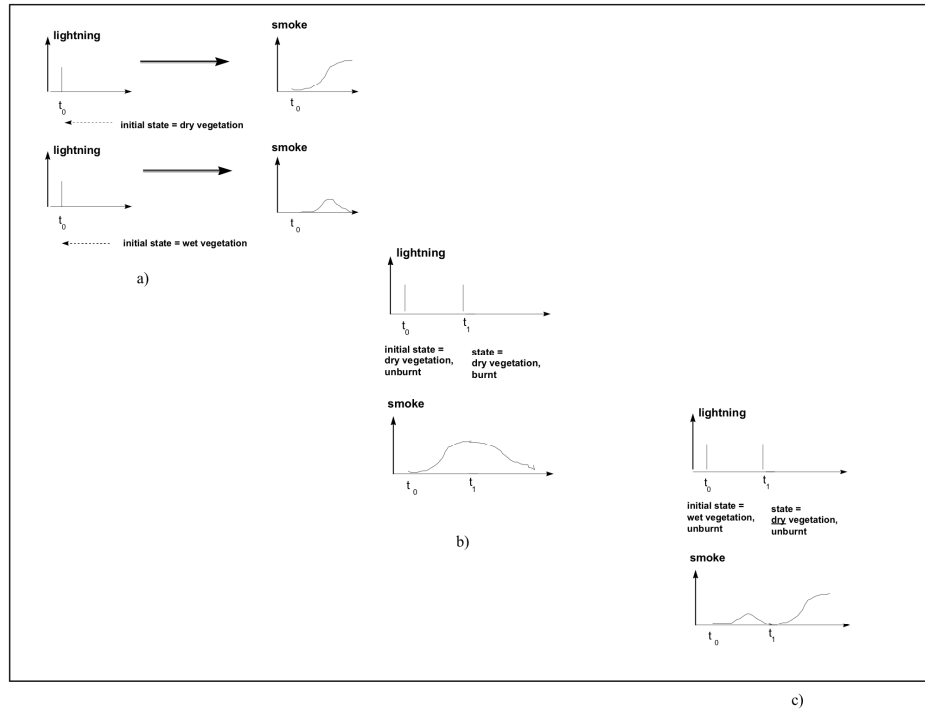
### 1.4.3 State Transition System Specification

At the next level (3) of system specification, we can specify not only initial state information but also how the *state changes* as the system responds to its input trajectory. Figure 13 b) and c) illustrate this important concept. Figure 13 b) presents the situation where the forest is in state (dry vegetation, unburned) when a lightning bolt occurs at time  $t_0$ . The state that the forest is in at time  $t_1$  when a second bolt occurs is (dry vegetation, burnt) reflecting the fact that a fire has ignited. Since the forest is in a different state, the effect of this second bolt is different from the first. Indeed, since there is little left to burn, there is no effect of the second bolt.

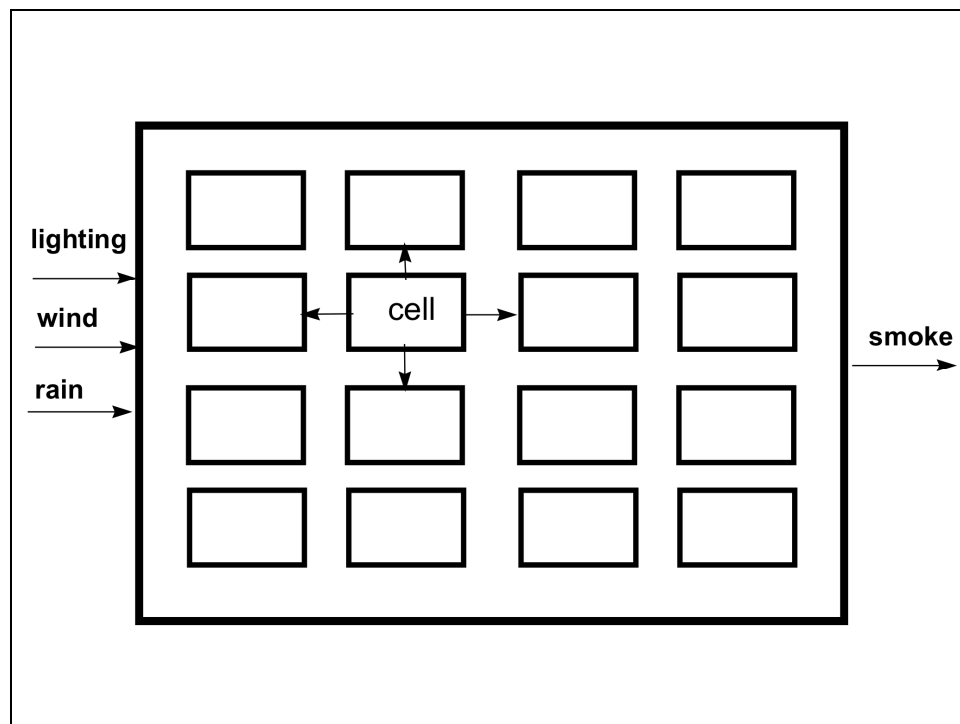
In contrast, Figure 13 c) illustrates the situation where the forest is wet and unburned when the first bolt occurs. It does not cause a major fire, but it does dry out the vegetation so the resulting state is (dry, unburned). Now the second bolt produces a major fire, just as the first bolt did in Figure 13 b) – since both the state and subsequent input trajectory are the same, the response of the system is the same.

### 1.4.4 Coupled Component System Specification

At the highest level of system specification, we can describe more about the internals of the system. Until now, it was a black box, at first observable only through I/O ports. Subsequently, we were able to peer inside to the extent of observing its state. Now, at level 4, we can specify how the system is composed of interacting components. For example, Figure 14 illustrates how a forest system could be composed of interacting cells, each representing a spatial region, with adjacent cells interconnected. The cells are modeled at level 3, i.e., their state transition and output generation definitions are used to act upon inputs, and generate outputs, respectively to and from, other cells. The cells are coupled together using ports. The output ports of one cell are coupled to the input ports of its neighbors.



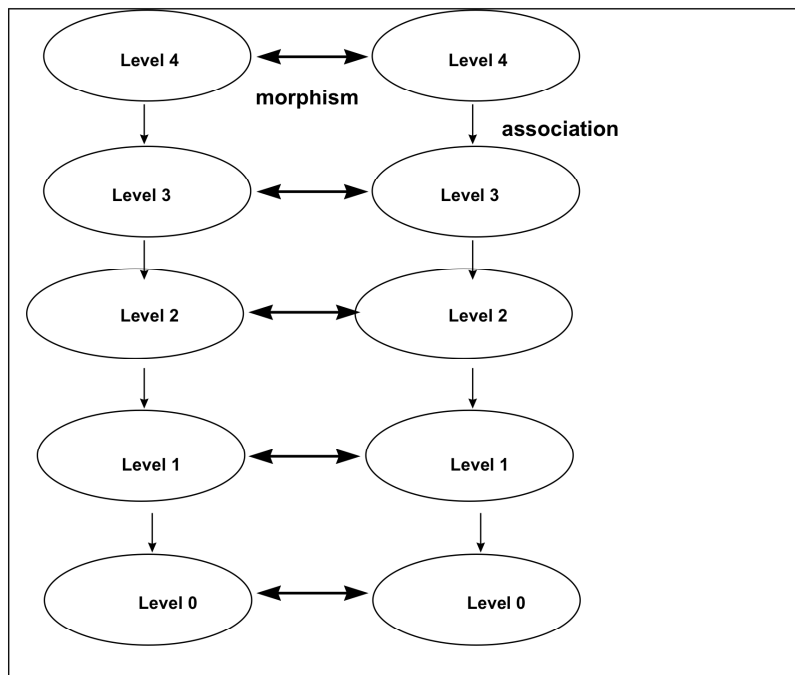
**Figure 13 Initial State Concept: a specification at Level 3 (I/O Function) in which we have initial state knowledge about the forest.**



**Figure 14 Component Structure System Specification for the Forrest System.**

## 1.5 System Specification Morphisms: Basic Concepts

The system specification hierarchy provides a stratification for constructing models. But, while constructing models is the basic activity in M&S, much of the real work involves establishing relationships between system descriptions. The system specification hierarchy also provides an orderly way of presenting and working with such relationships. Figure 15 illustrates the idea that pairs of system can be related by morphism relations at each level of the hierarchy. A morphism is a relation that places elements of system descriptions into correspondence as outlined in Table 4.



**Figure 15 Morphism Concepts for System Specification Hierarchy**

For example, at the lowest level, two Observation Frames are *morphic*, if we can place their defining elements – inputs, outputs, and time bases into correspondence. Such Frames are *isomorphic* if their inputs, outputs, and time bases respectively, are identical. In general, the concept of morphism tries to capture similarity between pairs of systems at the same level of specification. Such similarity concepts have to be consistent between levels. When we associate lower level specifications with their respective upper level ones, a morphism holding at the upper level must imply the existence of one at the lower level. The morphisms defined in Table 4 are set up to satisfy these constraints.

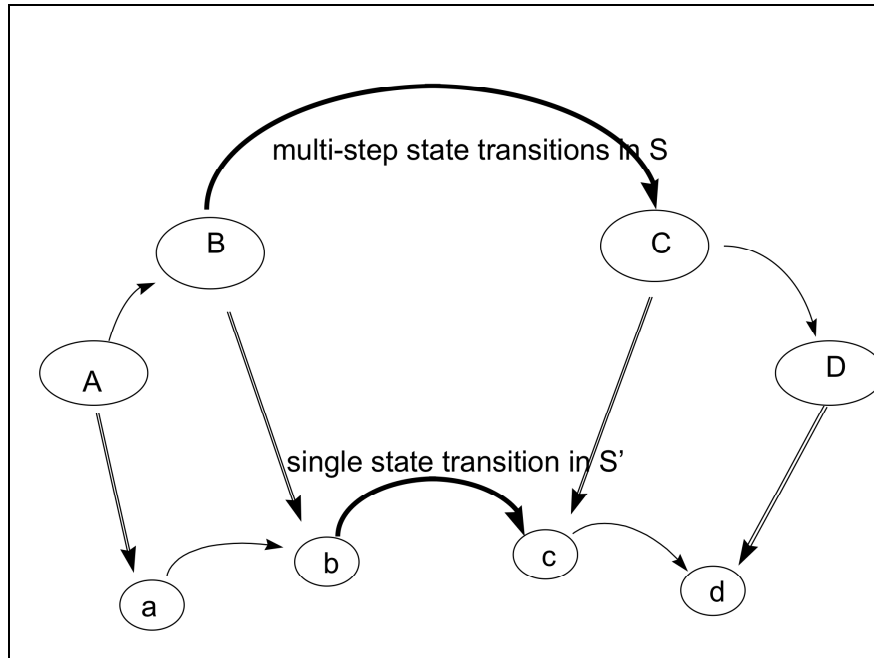
Level	Specification Name	Two Systems are Morphical at this level if:
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Level	Specification Name	Two Systems are Morphical at this level if:
0	Observation Frame	their inputs, outputs and time bases can be put into correspondence
1	I/O Behavior	they are morphic at level 0 and the time-indexed input/output pairs constituting their I/O behaviors also match up in one-one fashion
2	I/O Function	they are morphic at level 0 and their initial states can be placed into correspondence so that the I/O functions associated with corresponding states are the same
3	State Transition	the systems are homomorphic (explained below)
4	Coupled Component	components of the systems can be placed into correspondence so that corresponding components are morphic; in addition, the couplings among corresponding components are equal

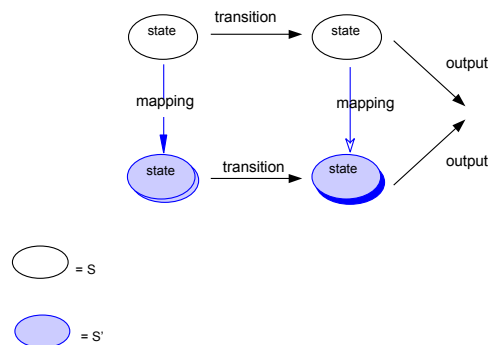
**Table 4 Morphism relations between systems in System Specification Hierarchy and Klir's levels.**

The most important morphism, called **homomorphism**, resides at the State Transition level and is illustrated in Figure 16. Consider two systems specified at level 3,  $S$  and  $S'$ , where  $S$  may be bigger than  $S'$  in the sense of having more states. Later, we'll see that  $S$  could represent a complex model and  $S'$  a simplification of it. Or  $S$  could represent a simulator and  $S'$  a model it is executing. When  $S'$  goes through a state sequence such as  $a, b, c, d$ , then  $S$  should go through a corresponding state sequence say  $A, B, C, D$ . Typically, a simulator has a lot of apparatus, represented in its states, necessary to accommodate the whole class of models rather than a single one. Thus we don't assume that states of  $S$  and  $S'$  are identical – only that there is a predefined correspondence between them illustrated by the shaded connecting lines in the figure. Now to establish that this correspondence is a homomorphism requires that whenever  $S'$  specifies a transition, such as from state  $b$  to state  $c$ , then  $S$  actually makes the transition involving corresponding states  $B$  and  $C$ . Typically, the simulator is designed to take a number of *microstate* transitions to make the *macrostate* transition from  $B$  to  $C$ . These are computation steps necessary to achieve the desired end result. It is not hard to see that if such a homomorphism holds for all states of  $S'$ , then any state trajectory in the  $S'$  will be properly reproduced in  $S$ .



**Figure 16 Homomorphism Concept.** This figure illustrates the preservation of state transitions that a homomorphism requires.

Often, we require that the correspondence hold in a step-by-step fashion. In other words, that the transition from a to b is mirrored by a one-step transition from A to B. Also, as just indicated, we want the I/O Behavior's of homomorphic models specified at the I/O System level to be the same. Thus, as in Figure 17, we require that the outputs produced from corresponding states be the same. In this type of homomorphism, the values and timing of the transitions and outputs of the base model are preserved in the lumped model. Thus, in this case, the state and output trajectories of the two models, when started in corresponding states, are the same.



**Figure 17 Homomorphism: a mapping preserving step-by-step state transition and output**

## 1.6 Summary

We have outlined the basic concepts of systems theory: structure, behavior, levels of system specification and their associated morphisms. We have brought out the important distinctions that justify having different levels of specification. However, we have not considered all the possible distinctions and levels. For example, the important distinction between modular and non-modular systems has not been recognized with distinct levels. A more complete hierarchy will emerge as we revisit the concepts introduced here in a more formal and rigorous manner in Chapter 5. We also have introduced the basic system specification formalisms and outlined the advances in the development of such formalisms that characterize this second edition, TMS98.

We now turn to a framework for modeling and simulation that identifies the key elements and their relationships. The systems specification hierarchy will provide the basis for presenting this framework. For example, we use specifications at different levels to characterize the different elements. The various system specification formalisms and their simulators provide the operational means to employ the framework in real world applications. We focus on real world application in the last part of the book.

## 1.7 Sources

The basic concepts of systems theory were developed by pioneers such as Arbib [Arbib 67], Zadeh [Zadeh 79] (later known more for his fuzzy sets theories), Klir [Klir 85], Mesarovic [Mesarovic 75] and Wymore. [Wymore 77]. Since the first edition of this book [Zeigler 76] there have been several trends – toward deepening the theory ([Mesarovic 89] [Wymore 93]), extending its range of applicability with computerized tools [Pichler 92] and going on to new more abstract formulations [Takahashi 95]. Also, somewhat independently, a new recognition of systems concepts within discrete event systems was fostered by Ho [Ho 94]. The DEV&DESS formalism was introduced by Praehofer in his doctoral dissertation [Praehofer 91]. The DEVS Bus originated in the research group of Tag Gon Kim [Yong 96]. Quantized system theory was first presented in [Zeigler 98]. A recent collection of systems concepts in computer science is given in [Allbrecht 98].

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## Definitions, Acronyms, Abbreviations

DEDS – Discrete Event Dynamic Systems

DESS – Differential Equation System Specification

DEVS – Discrete Event System Specification

DTSS – Discrete Time System Specification

DEV&DESS – Discrete Event and Differential Equation System Specification

M&S – Modeling and Simulation

TMS76 – 1976 Edition of Theory of Modeling and Simulation

TMS98 – 1998 Edition of Theory of Modeling and Simulation